Testing Purchasing Power Parity within a Co-Integration VAR-model Framework

Econometrics II – Assignment III

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1 Introduction

This paper investigates the empirical evidence for the purchasing power parity (PPP) using data for United Kingdom and the United States. PPP states that the exchange rate between two currencies should allow the same amount of goods to be purchased in either currency with the same amount of funds. PPP is a main result in neoclassical economics and theory of international monetary economics and therefore it is interesting to test the empirical foundation.

The analysis is conducted on data from 1872-2016, sub-dividing into two periods respectively 1872-1914 and 1915-2016. We use a Vector Autoregressive Model (VAR) to make a cointegration analysis. The results suggest that we cannot reject that the PPP theory holds as a long run equilibrium when we use the full sample size, but when dividing into sub-samples results are less clear. Finally we discuss our findings and make comparison to the results by Taylor (1988), Kim (1990) and Corbae (1988).

2 Economic Theory

2.1 Cointegration

Cointegration between variables can be thought of as defining an economic equilibrium, in the sense that the variables may wander arbitrarily due to stochastic trends, but never deviate too far from an equilibrium. Variables will not converge to some equilibrium point, but will have no inherent tendency to move away from equilibrium.

We say that variables in a vector $x_t \in \mathbb{R}^p$ cointegrate if there exists a cointegration vector β such that $\beta' x_t$ is stationary. For $p > 2$ there may be more than one cointegration vector (Nielsen, 2019).

PPP as an equilibrium We might consider PPP as a valid equilibrium, in which case $\beta = (1, -1)'$ and $\tilde{\beta} = (1, -1, -1)'$ are cointegrating vectors for x_t and $\tilde{x}_t$ respectively, see eq. 2.2 and eq. 2.3. If we, however, find βx_t or $\tilde{\beta} \tilde{x}_t$ to be non-stationary, we may not interpret PPP as a valid equilibrium, and might not declare PPP as empirically valid.

2.2 Econometric Model

In the following we will analyse cointegration using a Vector Autoregressive Model (VAR) framework. Compared to an ADL approach and a Engle-Granger two-step estimation approach we avoid the assumption of only one cointegration relationship between variables, something we cannot be sure of when conducting analyses on more than two variables as we do below. Further we do not pose any assumption of the direction of causality as in the ADL model.

Conceptually the VAR approach is more elegant, amongst other reasons due to handling of spurious regressions and unit root tests. The applied rank test procedure described in section 2.4 is a generalisation of the Dickey Fuller unit root test for the univariate case.

2.2.1 Model specification

We consider the following model for k lags:

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + \ldots + \Pi_k x_{t-k} + \mu + \epsilon_t, \quad t = 1{,}2, \ldots, T, \tag{2.1}$$

conditional on the initial values $x_0, x_{-1}, \ldots, x_{-(k-1)}$ and we assume $\epsilon_t \mid x_{t-1}, x_{t-2}$ to be identically and independently distributed. In the above we define

$$x_t = \begin{pmatrix} p_t - p_t^* \\ e_t \end{pmatrix} \tag{2.2}$$

But will later substitute x_t with

$$\tilde{x}_t = \begin{pmatrix} p_t \\ p_t^* \\ e_t \end{pmatrix} \tag{2.3}$$

Deterministic trends We allow for an unrestricted constant when estimating models, however leave out considerations on restricted trends or constant for later work, as they will lead to unnecessary complication of the model.

2.3 Error Correction

The notion of *error correction* or *equilibrium correction* simply stated means that whenever the observed variable deviates from its equilibrium value it will tend to return to its equilibrium value.

The *Granger Representation Theorem* states that any unit-root non-stationary variables are cointegrated if and only if there exists an error-correction model for either (subset of) variable(s) or all.

We might as such rewrite equation 2.1 for $k = 2$ lags as the vector error correction model:

$$\Delta x_t = \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \epsilon_t, \tag{2.4}$$

where $\alpha = (\alpha_1, \alpha_2)'$ (alternately $\alpha = (\alpha_1, \alpha_2, \alpha_3)'$ when using $\tilde{x}_t$) is the speed of adjustment and $\beta = (1, -\beta_2)'$ (alternately $\beta = (1, -\beta_2, -\beta_3)'$) is the cointegration vector.

2.3.1 Speed of adjustment

If the model error corrects, that is the variables cointegrate, we find that the magnitude of α may be referred to as the *speed of adjustment.* In each period a factor α will be removed from the deviation from equilibrium.

2.4 Testing rank of coefficient matrix

We may use the rank of the coefficient matrix $\Pi = \alpha\beta'$ to identify whether x_t is stationary or has unit roots, and if so whether the variables in x_t cointegrate.

We may determine the cointegration rank by looking at a series of nested models: $H_0 \subset H_1 \subset \ldots \subset H_r \ldots \subset H_p$, where subscript determine the rank of the coefficient matrix. We calculate the likelihood ratio (trace) statistics, the distributions of which are non-standard and dependent on the deterministic specification. We choose the smallest model not rejected.

Rank 0 If the coefficient matrix has rank 0 it is evidence that there is no stationary combination of the levels, that is the variables do not cointegrate, and equation 2.4 will only balance for $\Pi = 0$ which indicates that $\alpha = 0$ and there will be no error correction. The VAR-model will simplify to a simple VAR-model in first difference (as treated in assignment 2).

Full rank If the coefficient matrix has full rank (that is the rank is equal to the number of variables in x_t), x_t is a stationary process with no unit roots.

Reduced rank (not 0) If the rank of the coefficient matrix is between 0 and p, the full number of variables in x_t, we find that x_t is unit-root non-stationary and the variables in x_t cointegrate. The strength of error correction is given by α.

2.5 Tests for misspecification

We may test for misspecification, using the tests listed in table 1.

Applied Test:	Null:	Test-statistic
LM(1)-LM(2)	No autocorrelation	χ^2
Jarque-Bera	No significant deviation from normally distributed error terms	χ^2

Table 1: Misspecification tests

Most importantly the model may not suffer from autocorrelation, as this imply that estimates will be inconsistent. Normality ensures that estimates converge quickly to the true value. Heteroscedasticity violates our stationarity assumption of constant variance, however may be circumvented by using robust errors.

2.5.1 Test for lag determination

In order to determine the number of lags we use LR-testing to identify if imposed restriction are acceptable. If the null hypothesis given by imposed restriction is true, the test will be distributed:

$$LR(k = j - \frac{i}{p} \mid k = j) \overset{\mathrm{d}}{=} \chi^2(i), \tag{2.5}$$

where j is the number of lags in the unrestricted model, i is the number of imposed restriction and p is the number of dimensions in the vector Z_t.

3 Description of Data

Data cover the period from 1870 to 2016, and it is taken from Jordà-Schularick-Taylor macro-historic database. We define the variables:

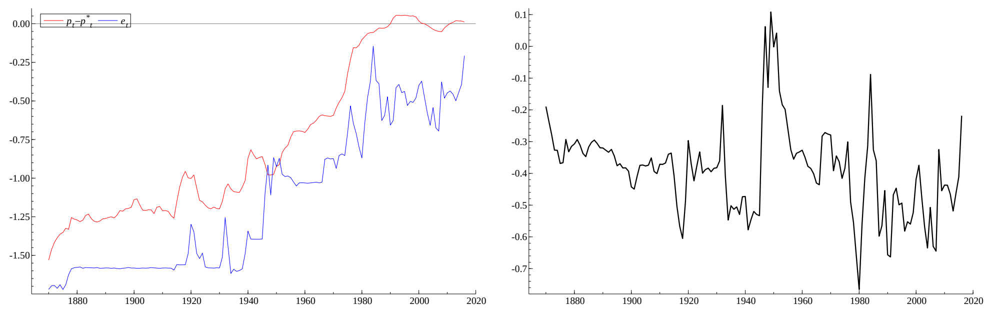
$$p_t = \log \mathsf{UKcpi}_t \quad (3.1)$$

$$p_t^* = \log \mathsf{UScpi}_t \quad (3.2)$$

$$e_t = \log \mathsf{Sterling}_t \quad (3.3)$$

where UKcpi_t is the consumer price index in United Kingdom, UScpi_t is the consumer price index in the United States, both with 1990=100, and where $\mathsf{Sterling}_t$ is the bilateral exchange rate denominated as pounds per dollar. The exchange rate and the difference between price levels are plotted in figure 1b.

Figure 1: Graphical representation of relevant measures



(a) Exchange rate, e_t, and difference in consumer price index for UK and US

(b) Power Purchasing Parity: $e_t - (p_t - p_t^*)$, 1870-2016

3.1 Shifting monetary regimes

Over the course of the sample period we identify four different monetary regimes: 1) Gold Standard [1870-1918], 2) Interbellum years, floating exchange rate [1919-1945], 3) Bretton Woods [1946-1971] and 4) Floating exchange rate [1972-2016].

It might be interesting to examine each sub-period by themselves, however this would require high-frequency data, as we would face sample size restrictions using data at hand.

We thus split in two samples 1872-1914 (fixed rate) and 1915-2016 (floating rate). We exclude WWI from the first sample due to large fluctuations in this period. The sub-periods are further discussed in section 5.

4 Empirical model

We examine two sub-periods respectively 1870-1914 (period I) and 1915-2016 (period II) and the full sample period for the two models, given by eq. 2.2 (model 1) and eq. 2.3 (model 2).

4.1 Misspecification

Dependent on misspecification test results we find different lag-lengths depending on model and sample size in order to obtain well-specified models. The used number of lagged variables (k) are shown in table 2. Correct determination of lag length is of importance as too few lags lead to rejection of the null too easily and too many lags decrease the power of the tests (Verbeek, 2017).

Table 2: Overview of used lag-length

	Full sample size: 1870-2016	First period: 1870-1914	Second period: 1919-2016
Model 1	2	2	3
Model 2	4	2	3

Notes: For model 1(2) the two(four) first observations are used as initial values. Model 1 refers to equation 2.2 and model 2 refers to eq. 2.3. Number of used lags refer to k in eq. 2.1

In table 3 we report results from misspecification test on the models we later show have reduced rank, which turns out to be either model for the full sample. We note that models (in general) do not fulfill normality of error terms, in which case we estimate using Quasi Maximum Likelihood (QMLE).

Table 3: Misspecification test results for model 2.2 and 2.3, 1870-2016

	Autocorrelation		Normality
	LM(1) (χ^2)	LM(2) (χ^2)	
Model 1 (2 lags)	6,3815	8,3298	55,858
	(0,1724)	(0,0802)	(0,0000)
Model 2 (4 lags)	9,3525	12,110	141,87
	(0,4054)	(0,2072)	(0,0000)

Notes: Numbers in parentheses are p-values. For model 1(2) the two(four) first observations are used as initial values. Model 1 and Model 2 refer to equation 2.2 and 2.3 respectively.

4.2 Rank test

4.2.1 Model 1

We find that the coefficient matrix for model 1 in period I has full rank. This means that x_t is a stationary process and there is no unit-root, graphically see first part of figure 1b.

In period II the coefficient matrix for model 1 has no rank, which means that there is a unit-root non-stationary process and the series are not cointegrated.

For the whole sample period, the coefficient matrix of model 1 is of rank 1. This implies a unit-root non-stationary process, but in relation to the results above the variables of x_t are cointegrated.

4.2.2 Model 2

The coefficient matrix for model 2 has no rank for both sub-periods. Interestingly, for the whole sample model 2 has rank 1. This means x_t is a non-stationary process with a unit-root and the series are cointegrated, thus $\tilde{\beta}'\tilde{x}_t$ is stationary.

For both sub-samples we cannot confirm that PPP holds as a valid equilibrium relation, regardless of applied model. We will further investigate whether the models under full sample size hold under the PPP theory.

4.3 Estimation results

The models are denoted in the following:

$$\Delta x_t = \begin{pmatrix} \underset{(0.0201)}{0.034} \\ \underset{(0.0605)}{-0.241} \end{pmatrix} \begin{pmatrix} \underset{(0.0507)}{-0.921} & \underset{(-)}{1} \end{pmatrix} \begin{pmatrix} p_{t-1} - p^*_{t-1} \\ e_{t-1} \end{pmatrix} + ... \tag{4.1}$$

$$\Delta \tilde{x}_t = \begin{pmatrix} \underset{(0.0235)}{0.05} \\ \underset{(0.0223)}{0.0391} \\ \underset{(0.0618)}{-0.262} \end{pmatrix} \begin{pmatrix} \underset{(0.232)}{-0.0969} & \underset{(0.334)}{-0.24} & \underset{(-)}{1} \end{pmatrix} \begin{pmatrix} p_{t-1} \\ p^*_{t-1} \\ e_{t-1} \end{pmatrix} + ... \tag{4.2}$$

Notes: Numbers in parentheses are standard errors. Eq. 4.1 and eq. 4.2 refer to model 1 and model 2, respectively.

As we are aware that $\beta' x_t$ is stationary we may write:

$$\begin{aligned} & -0.921(p_{t-1} - p^*_{t-1}) + e_{t-1} = u_t, \;\; u_t \sim I(0) \\ \Leftrightarrow \;\; & e_{t-1} = 0.921(p_{t-1} - p^*_{t-1}) + u_t, \end{aligned} \tag{4.3}$$

which can be interpreted in the way that of the price split increases 10 pct. the nominal exchange rate increases 9.21 pct., that is an almost 1 to 1 increase (below we fail to reject a one to one relation). We find that the nominal exchange rate error corrects. We know that this is contrary to reality during fixed exchange rate regimes. We do not find significant error correction for the price split (using a $\chi^2(1)$ distribution). We further note that using model 2 the β coefficients are not significant.

4.4 Imposing restrictions according to PPP

We are testing if the PPP holds as a long run equilibrium relation. We are imposing restrictions on the beta-values for both models that cover the full sample period, and formulate the null hypotheses as follows:

Model 1, 1870-2016:

$$H_0 : \beta = (1, -1) \;\; \text{and} \;\; H_A : \beta \neq (1, -1)$$

Model 2, 1870-2016:

$$H_0 : \tilde{\beta} = (1, -1, -1) \;\; \text{and} \;\; H_A : \tilde{\beta} \neq (1, -1, -1)$$

In both cases we cannot reject that PPP holds as a long run equilibrium relation on the basis of the χ^2 results, shown in table 2.

5 Discussion and concluding notes

5.1 Sample size

Results are generally very sensitive to changes in sampling. For instance, including 1914-1918 (the WWI years) in the first sample, results in rank(Π) = 0, opposed to excluding the period, which results in the coefficient matrix having full rank.

Table 4: Results from imposing restrictions on β-values

	χ^2
Model 1	1,9191 (0,1660)
Model 2	0,89154 (0,3451)

Notes: Numbers in parentheses are standard errors.

In turn we only find evidence of co-integration when covering the whole sample period, but not when looking at subsamples (see 4.2.2). This may most probably be an issue with lacking variability in data. Further we do not find significant results when estimating model 2.

5.2 PPP theory

We have attempted to examine empirical evidence for PPP as a long run principle. Based on the data at hand we get mixed results. Based on model 1 for the full sample we cannot reject that the PPP is a valid equilibrium when using the full sample, however when applying the model to shorter periods, we do not find evidence of co-integration. We cannot reject that this is an issue of lacking variability in data, only.

5.2.1 Comparison with litterature

Results are generally in line with (Kim, 1990), in the way we find PPP might hold in general, however when looking at other periods, such as later periods with fluent exchange rate we do not find evidence for PPP. However as Kim and we previously have pointed to, this might be an issue of limited variability in data.

One could suggest using data with higher frequency, however Taylor does not find evidence for cointegration despite using monthly data.

5.3 Differences between Model 1 og Model 2

We would expect to find similar results using both Model 1 and Model 2, as we note that $(1,-1)(p_t - p_t^*,\ e_t)' = (1,-1,-1)(p_t,\, p_t^*,\, e_t)'$ However when estimating we only find significant results for Model 1, which might be a sign of lacking variability in data.

In sum, results are not stricly clear for or against PPP, however we could argue that the PPP can be a relevant equilibrium for the whole period in the long run perspective.